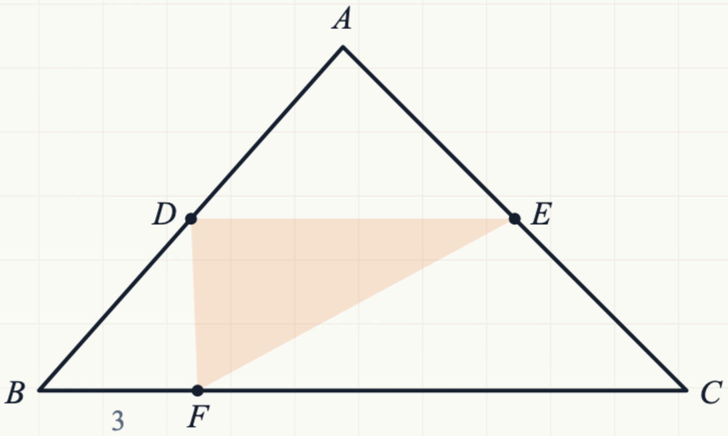
THE PROBLEM WE WILL CRACK TODAY

Triangle ABC has area 16. D and E are midpoints of AB and AC. F is on BC with BF = 3.
What is the area of triangle DEF?



→ area of ABC = 16

→ D, E are midpoints

→ BF = 3

= ?

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International school student · strong in maths, and asks the questions everyone else is thinking

Here as a **student**, not a second teacher

Chapter 1 — Perimeter and Area

1. **Lessons 1–5** — the ideas, taught through Examples 1 to 28
2. **Lessons 6–8** — the homework problems, worked in full
3. Today: triangles. Perimeter, area, equilateral, and Heron
4. Every lesson ends with a puzzle. The answer comes at the start of the next one

Eight lessons, about thirty minutes each.

Area is not a formula. It is a choice.

Almost every triangle question is decided by **which base you pick**.
Pick well and the problem is one line. Pick badly and it is ten.

Keep this in mind for the whole lesson.

Lengths

l — length

w — width

h — height

r — *radius*

Totals

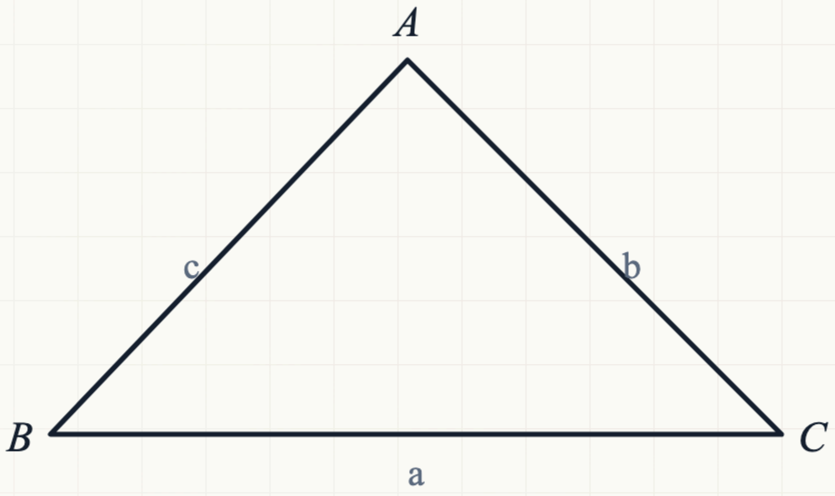
P — perimeter

A or **S** — area

C — circumference

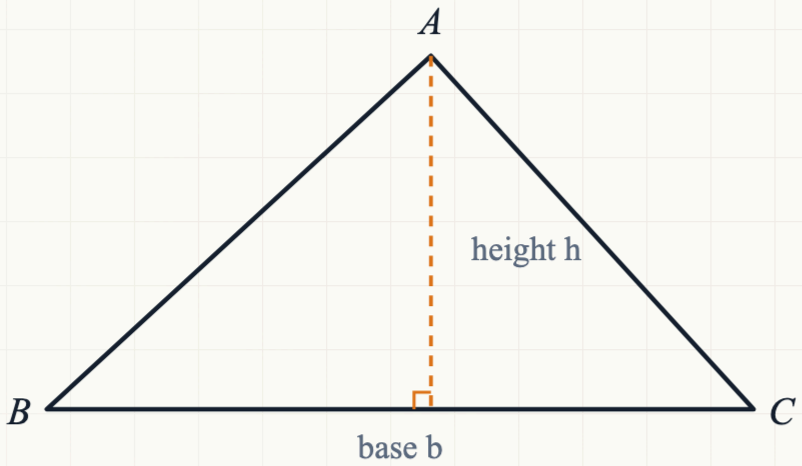
a, b, c — *side lengths*

Perimeter of a triangle — the distance all the way around



→ $P = a + b + c$

Area of a triangle — and the word that trips everyone up



→ $A = \frac{1}{2} \times b \times h$

→ h is measured ⊥ to b — straight down, not along a side

Base and height come as a matched pair.

You may pick **any** of the three sides as the base.

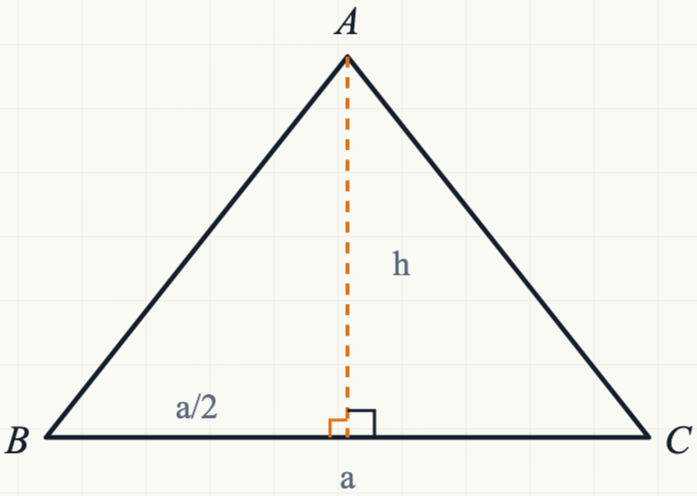
But then the height must be the one measured to **that** side.

Three different pairs. Three identical answers. Pick the easy pair.

Your turn — pause and answer

1	base 10, height 6 → area?	30
2	area 30, base 12 → height?	5
3	right triangle, legs 7 and 8 → area?	28

Equilateral triangle — where the formula comes from



→ the height cuts the base exactly in half

→ $h^2 + (a/2)^2 = a^2 \rightarrow h = (\sqrt{3}/2)\,a$

→ $A = \frac{1}{2} \cdot a \cdot (\sqrt{3}/2)\,a$

$$\boxed{A = (\sqrt{3}/4)\,a^2}$$

Your turn — an equilateral triangle with side 6

1. Using $A = (\sqrt{3}/4)a^2$ **$9\sqrt{3}$**
2. Or: find the height first, then $\frac{1}{2}bh$ **same answer**

Heron's formula — when you only know the three sides

$s = (a + b + c) / 2 \leftarrow$ the semi-perimeter

$$A = \sqrt{[\, s(s - a)(s - b)(s - c) \,]}$$

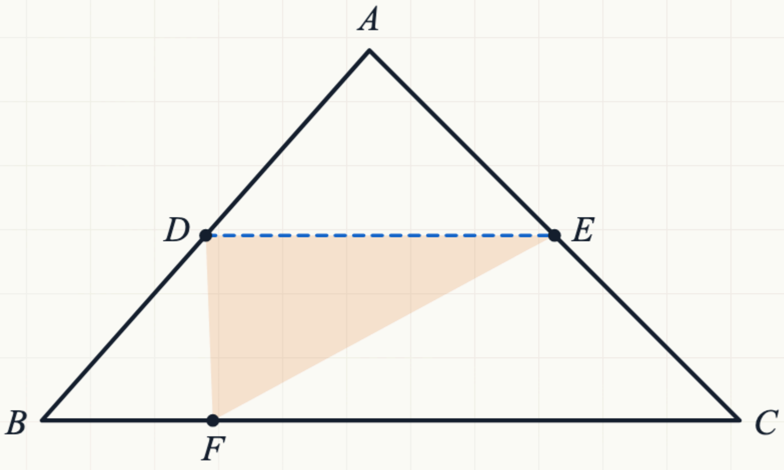
No height needed. Three sides in, area out.

Your turn — sides 9, 12, 15

1	Check first: is it a right triangle?	yes — 3 × (3,4,5)
2	So the area is…	54
3	Heron gives the same, in four times the work	54

EXAMPLE 1

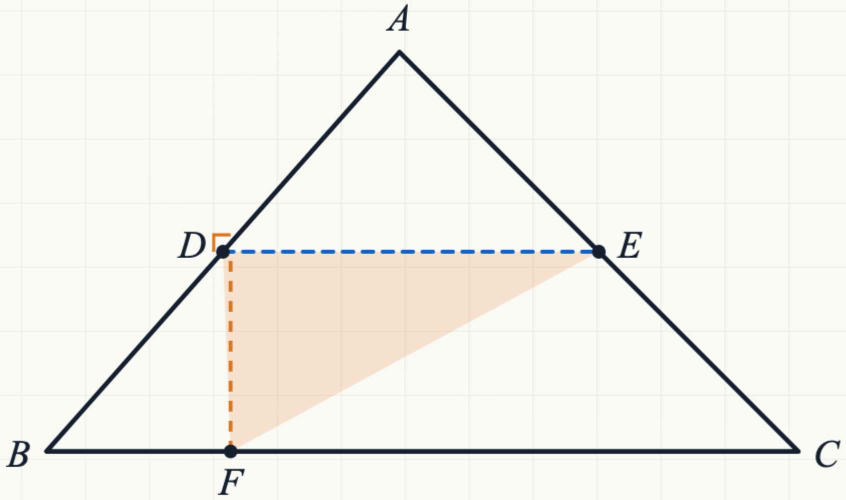
Area of $ABC = 16$. D, E are midpoints of AB, AC. F is on BC, $BF = 3$. Find the area of DEF.



→ (A) 12 (B) 9 (C) 8 (D) 4 (E) 3

EXAMPLE 1

Take *DE* as the base. Now find the height from *F*.



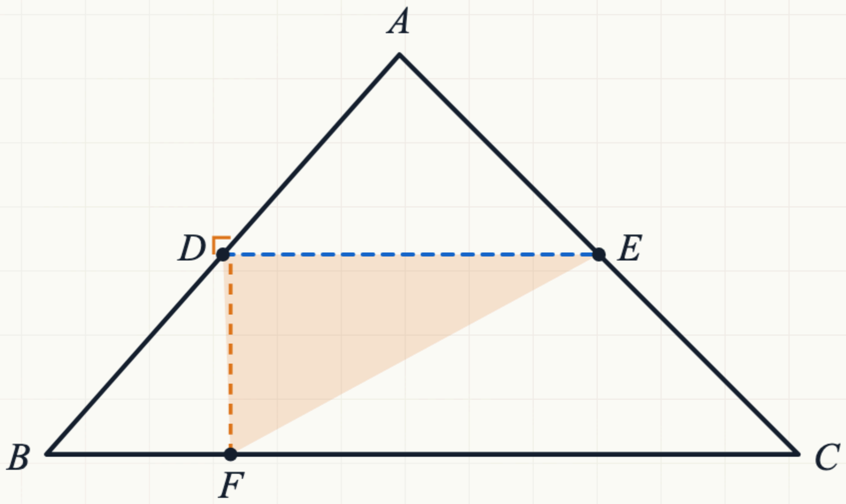
→ DE is halfway up the triangle

→ so the gap from DE down to BC is half the full height

→ F sits on BC → height of DEF = ½h

EXAMPLE 1

Put the two halves together.



→ base $DE = \frac{1}{2} \cdot BC$

→ height $= \frac{1}{2} \cdot h$

→ area $DEF = \frac{1}{2} \cdot (\frac{1}{2} BC) \cdot (\frac{1}{2} h) = \frac{1}{4} \cdot (\frac{1}{2} BC \cdot h)$

→ $= \frac{1}{4} \times 16$

(D) 4

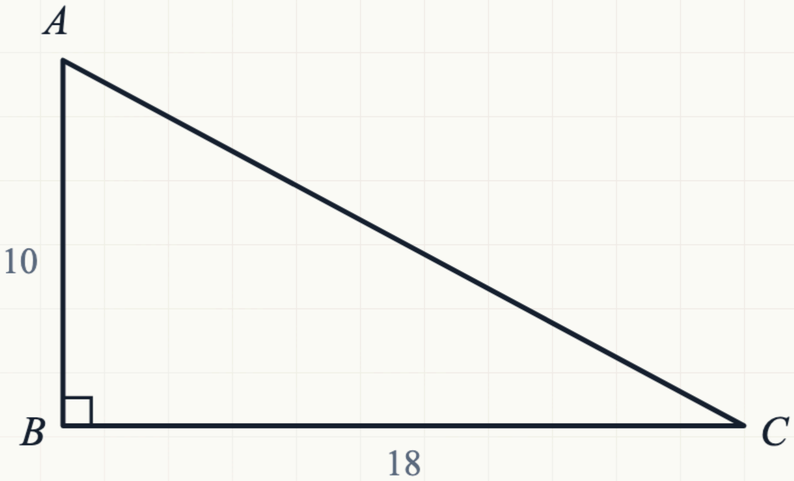
Not every number in a problem is for you.

AMC problems contain numbers that exist only to see whether you know which numbers you actually need.

Before you use a value, ask what it is doing in the problem.

EXAMPLE 2

What is the area of a right triangle whose legs measure 10 cm and 18 cm?



→ (A) 180 (B) 100 (C) 90 (D) 60 (E) 30

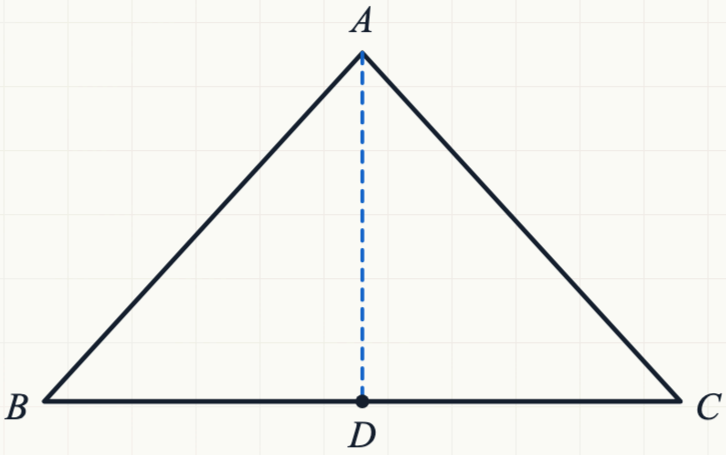
→ in a right triangle the two legs are already ⊥

→ $A = \frac{1}{2} \times 10 \times 18$

(C) 90 cm²

EXAMPLE 3

$BD = DC$, and the area of triangle ABD is 8 cm².
Find the area of triangle ABC.



→ (A) 16 (B) 8 (C) 14 (D) 32 (E) 4

Equal base + equal height = equal area.

The two triangles do not have to look alike.

A median always cuts a triangle into two **equal** areas.

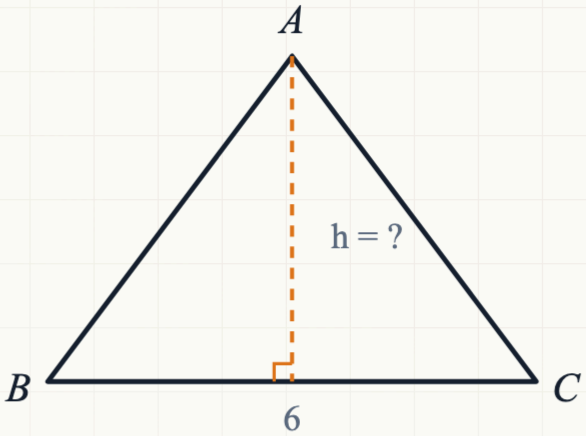
This one fact solves a surprising number of shaded-area problems.

Your turn — the median again

1. A median splits a triangle. One half has area 7. The whole? **14**
2. Does the shape of the two halves matter? **no**

EXAMPLE 4

A triangle has area 24 square units and base 6 units. How long is the height?



→ (A) 24 (B) 12 (C) 10 (D) 8 (E) 6

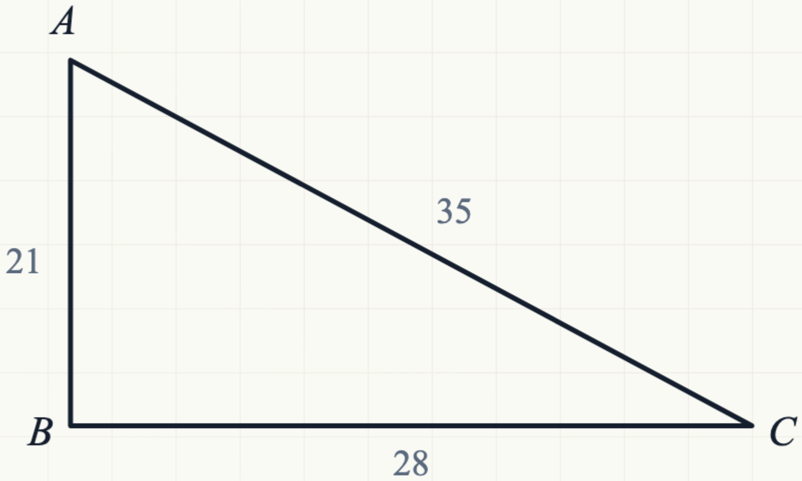
→ $24 = \frac{1}{2} \times 6 \times h$

→ $24 = 3h$

(D) 8

EXAMPLE 5

Find the area of the triangle with side lengths 21, 28 and 35.

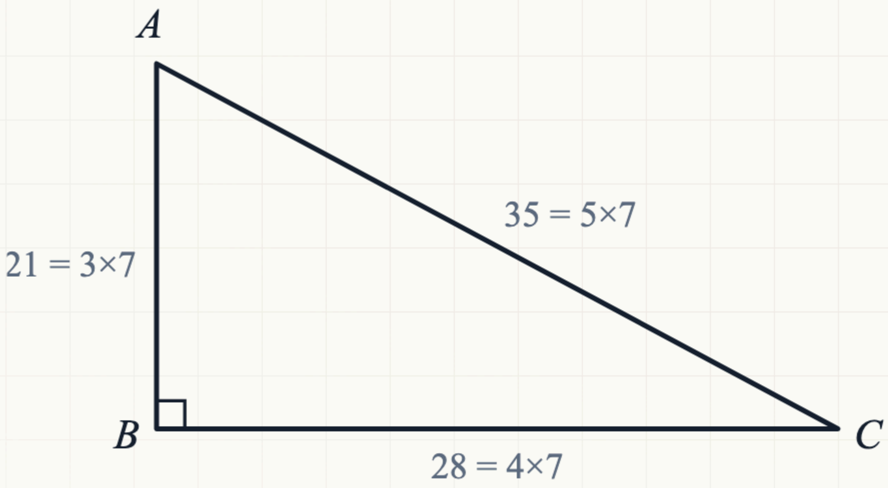


→ (A) 84 (B) 42 (C) 144 (D) 288 (E) 294

→ three sides, no height given

EXAMPLE 5 — THE FAST WAY

Look at the three numbers again: 21, 28, 35



→ $21 : 28 : 35 = 3 : 4 : 5$

→ so it is a right triangle — legs 21 and 28

→ $A = \frac{1}{2} \times 21 \times 28$

(E) 294

Last one — sides 10, 24, 26

1	Common factor?	**2 → 5, 12, 13**
2	Right triangle?	**yes**
3	Area?	**120**

Four tools, and one habit

1. **$P = a + b + c$** — and read what the question actually wants
2. **$A = \frac{1}{2}bh$** — the height is **perpendicular** to the base you chose
3. **$A = (\sqrt{3}/4)a^2$** — equilateral; rebuild it from the half-triangle if you forget
4. **Heron** — three sides, no height. But check for a right triangle first

PUZZLE — ANSWER AT THE START OF LESSON 2

The farmer's field

A farmer's field is a triangle with sides **6 m**, **8 m** and **10 m**. He wants one straight fence from a corner that splits the field into two equal areas.
Where should it go — and why does it work from **any** corner?

Work it out before Lesson 2. Hint: you already proved this today.